377. Proposed by W. D. CAIRNS, Oberlin College.

It is required to find a curve of the form $y = x(x - a)(x - b)$ such that the abscissas of the maximum and minimum values, as well as a and b, shall be positive integers.

MECHANICS.

When this issue was made up, no solutions had been received for numbers 289, 292–3, 295–299, 301, and 303.

302. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

A ball is projected from a given point at a given inclination β towards a vertical wall; determine the velocity so that after striking the wall the ball may return to the point of projection.

NUMBER THEORY.

When this issue was made up, no solutions bad been received for numbers 215–16, 218, 220, and 223–226.

226. Proposed by ELBERT H. CLARKE, Purdue University.

If 0! is taken equal to 1, and if k is any positive integer greater than or equal to 2, show that

$$\sum_{n=0}^{\infty} \frac{n!}{(k+n)!} = \frac{1}{(k-1)} \cdot \frac{1}{(k-1)!}.$$

227. Proposed by R. P. BAKER, University of Iowa.

Show that every rational number can be expressed as a finite sum $\sum_{n=m}^{n=m+k} \frac{a_n}{n}$, where a_n is either 0 or 1 and m is any positive integer.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

409. Proposed by C. E. GITHENS, Wheeling, W. Va.

Find integral values for the edges of a rectangular parallelopiped so that its diagonal shall be rational.

II. SOLUTION BY ARTEMAS MARTIN, Washington, D. C.

On pp. 269–273 of the October, 1914, MONTHLY, W. C. Eells has solved a different problem from the one proposed. In making $x^2 + y^2 = \square$, he adds another condition not required.

Let x, y, z be the edges and d the diagonal of the parallelepiped; then we have to satisfy the equation

$$x^2 + y^2 + z^2 = d^2.$$

It is not necessary that $x^2 + y^2$ be a square. Let us assume $x = a$, $y = b$, $z + c = d$, and we have

$$a^2 + b^2 + z^2 = (z + c)^2 = z^2 + 2cz + c^2,$$

which immediately gives

$$z = \frac{a^2 + b^2 - c^2}{2c} \qquad \text{and} \qquad d = \frac{a^2 + b^2 + c^2}{2c}.$$

Therefore

$$a^2 + b^2 + \left(\frac{a^2 + b^2 - c^2}{2c}\right)^2 = \left(\frac{a^2 + b^2 + c^2}{2c}\right)^2,$$

whatever be the values of a, b, c.

1. Taking $a = 1$, $b = 2$, $c = 1$, we find

$$1^2 + 2^2 + 2^2 = 3^2,$$

which is the *smallest* rational parallelepiped.

2. Taking $a = 2$, $b = 3$, $c = 1$, we have

$$2^2 + 3^2 + 6^2 = 7^2,$$

which is the smallest rational parallelepiped having its edges all different.

3. Taking $a = 1$, $b = 4$, $c = 1$, we get

$$1^2 + 4^2 + 8^2 = 9^2.$$

4. Let $a = 2$, $b = 6$, $c = 2$, and we have

$$2^2 + 6^2 + 9^2 = 11^2.$$

5. If $a = 3$, $b = 4$, $c = 1$, we get

$$3^2 + 4^2 + 12^2 = 13^2,$$

which, on p. 269, is stated to be "the smallest rational parallelopiped."

6. If we take $a = 8$, $b = 9$, $c = 5$, we will get

$$8^2 + 9^2 + 12^2 = 17^2.$$

7. Taking $a = 4$, $b = 8$, $c = 2$, we have

$$4^2 + 8^2 + 19^2 = 21^2.$$

8. If $a = 3$, $b = 4$, $c = 3$, then we get

$$12^2 + 15^2 + 16^2 = 25^2.$$

And so on, there being an infinite number of parallelopipeds whose edges and solid diagonals are rational integers.

The condition $x^2 + y^2 + z^2 = \square$ can be satisfied in many ways.—See *Mathematical Magazine,* Vol. II., No. 5 (October, 1891), pp. 71–74. Other methods are given in a forthcoming paper which will appear in the third part of No. 12, Vol. II., of the *Mathematical Magazine.*

416A. Proposed by H. O. HANSON, East Elmhurst, N. Y.

Find the nth term and the sum of n terms of the series obeying the relation $u_i = u_{i-1} + 2u_{i-2}$ in terms of n and the first two terms, u_1 and u_2, these two terms being arbitrary.

This problem was incorrectly numbered 416.